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Fuad Aleskerov & Sergey Shvydun

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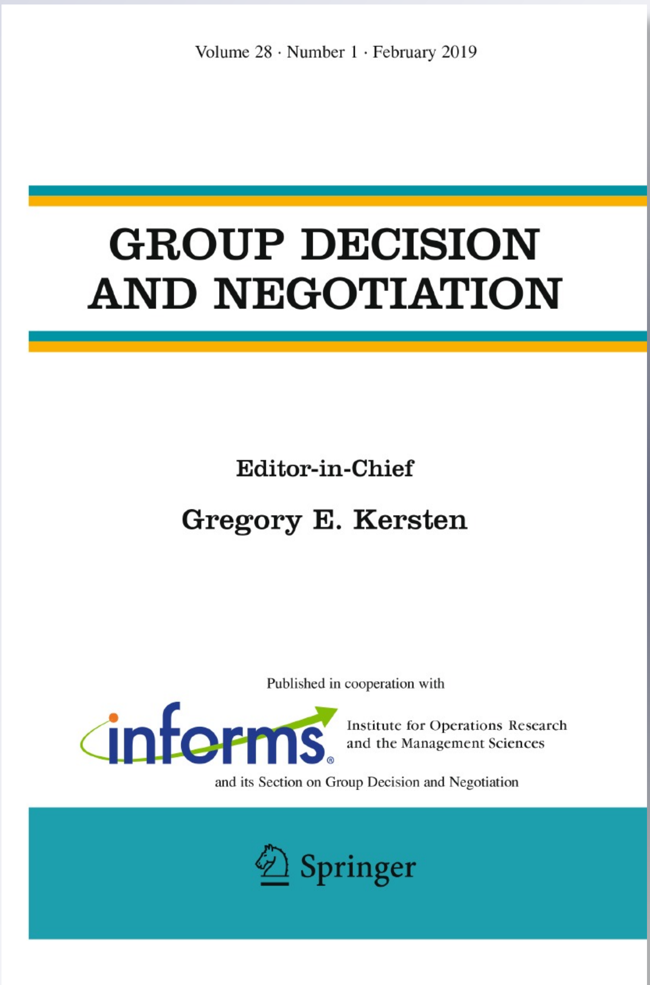

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Allocation of Disputable Zones in the Arctic Region

Fuad Aleskerov[1] · **Sergey Shvydun**[1]

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Abstract

As a result of the climate change the situation in Arctic area leads to several important consequences. On the one hand, fossil fuels can be exploited much easier than before. On the other hand, their excavation leads to serious potential threats to fishing by changing natural habitats which in turn creates serious damage to the countries' economies. Another set of problems arises due to the extension of navigable season for shipping routes. Thus, there are already discussions on how should resources be allocated among countries. In Aleskerov and Victorova (An analysis of potential conflict zones in the Arctic Region, HSE Publishing House, Moscow, 2015) a model was presented analyzing preferences of the countries interested in natural resources and revealing potential conflicts among them. We present several areas allocation models based on different preferences over resources among interested countries. As a result, we constructed several allocations where areas are assigned to countries with respect to the distance or the total interest, or according to the procedure which is counterpart of the Adjusted Winner procedure. We consider this work as an attempt to help decision-making authorities in their complex work on adjusting preferences and conducting negotiations in the Arctic zone. We would like to emphasize that these models can be easily extended to larger number of parameters, to the case when some areas for some reasons should be excluded from consideration, to the case with 'weighted' preferences with respect to some parameters. And we strongly believe that such models and evaluations based on them can be helpful for the process of corresponding decision making.

Keywords Arctic region · Oil and gas deposits · Fishing resources · Arctic shipping · Zones of mutual interests · Areas allocation

✉ Fuad Aleskerov
alesk@hse.ru

Sergey Shvydun
shvydun@hse.ru

1 National Research University Higher School of Economics, V.A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences, Moscow, Russia

1 Introduction

Arctic region has been a matter of intense disputes for the last several decades. Although it encompasses approximately 6% of the globe, the Arctic contains an estimated 30% of the world's undiscovered gas and 13% of the world's undiscovered oil (Gautier et al. 2009). It is further estimated that approximately 84% of these resources is located on offshore continental shelves (Bird et al. 2008). Due to extreme climatic conditions and, consequently, high costs, exploration and development of natural resources in the Arctic looked unattractive several decades ago. The recent changes in climate, resulting in ice melting, opened up new opportunities for the region. The Arctic waters became more accessible for oil and gas exploration and exploitation while the interest in north natural resources has also arisen due to increased global demand for energy. Offshore oil and gas activities have several potential consequences on the environment in the region (Koivurova and Hossain 2008). The fossil fuels exploration increases the pollution in the region and the likelihood of an oil spill, thus, affecting whole ecosystems. Another consequence is that exploration activities may also threaten the human security of the local and indigenous communities, their cultures, health and traditional livelihoods and create serious damage to the economies of the countries in the region. These facts make Arctic region as one of the most sensitive and vulnerable to climate change.

Besides energy resources, the Arctic is rich of other natural resources. In particular, there are 450 species of fish in the region (Arctic Council 2001). The changes in climate led to the fact that fishing seasons were also considerably extended by increased periods of open water (The European Science Foundation 2014) which may also lead to the problem of overfishing. Arctic shipping routes, which potentially may offer significant economic savings for many countries, also became more accessible. The potential economic benefits of global warming in the Arctic have already captured the interest of many countries and may result in potential intersection of mutual interests among them. Unclear borders and territorial claims made the problem even more complicated. This fact resulted in territorial claims and many disputes on how to determine who has the right to Arctic resources.

The Arctic Council which was established in 1996 is one of the effective forums for international cooperation and sophisticated political discussions on shared concerns in the Arctic region. It is comprised of eight Arctic states, twelve other states with an observer status and multiple non-state organizations. The council focuses on environmental protection, sustainable development, territorial claims and other issues of common importance to the Arctic nations (Arctic Council 2017). However, there are still some tensions over territorial borders and rights for natural resources among interested countries.

In Aleskerov and Victorova (2015) there was an attempt to estimate the utility for the interested parties of each area in the Arctic region. As a result, there was proposed a model which analyzes preferences over different zones of the countries interested in the Arctic region and reveals potential intersection of mutual interests among them. Several scenarios were discussed in which countries have different levels of interests in gas and oil deposits located in areas of other countries in order to identify zones of the highest level of mutual interests in the Arctic region. However, the model proposed in Aleskerov and Victorova (2015) considered the whole region and did not take into account the actual international boundaries between countries. Moreover, it did not propose any solution that allows to decrease the level of tension in the region. Our work extends that study and focuses on the intersection of mutual interests in the Arctic. We present several models of areas allocation based on the preferences over the main resources which may decrease the potential conflict of mutual interests.

The paper is organized as follows. In Sect. 2 we provide some information about territorial boundaries in the Arctic region and territorial claims of interested countries. In Sect. 3 we describe the methodology of our research and the data used. In Sect. 4 we propose several allocation models which allocate each zone of the Arctic to a particular country with respect to main resources—oil, gas, fish and maritime routes. In Sect. 5 we discuss the shared allocation of territories among countries. In these Sects. 4 and 5 we evaluate the efficiency of each model. Section 6 concludes.

2 International Boundaries and Territorial Claims in the Arctic

Generally, the Arctic region is shared by eight Arctic states—the United States of America (Alaska), Canada, Finland, Denmark (by way of Greenland), Iceland, Norway, Russia, and Sweden. Arctic lands and internal waters are governed by the laws of each Arctic state. The rights and responsibilities of states in their use of the world's oceans and the management of marine natural resources are defined in the United Nations Convention on the Law of the Sea (UNCLOS) which was signed and ratified by 157 and 166 states, respectively (UNCLOS 1982). According to it, "the sovereignty of a coastal state extends, beyond its land territory and internal waters and, in the case of an archipelagic State, its archipelagic waters, to an adjacent belt of sea, described as the territorial sea and limited to 12 nautical miles (approximately 22.2 km) from the baseline which is a low-water line along the coast. In the territorial sea the coastal state is free to set rules and use any resource while vessels of other states are given the right of passage if it is not prejudicial to the peace, good order or security of the coastal state".

In a zone contiguous to its territorial sea, described as the contiguous zone and limited to 24 nautical miles from the baseline, the coastal state "may exercise the control necessary to prevent or punish infringement of its customs, fiscal, immigration or sanitary laws and regulations within its territory or territorial sea" (UNCLOS 1982).

To define sovereign rights "for the purpose of exploring and exploiting, conserving and managing, the living and the non-living resources of areas outside territorial sea", a concept of an exclusive economic zone was proposed in the UNCLOS. According to (UNCLOS 1982), the exclusive economic zone (EEZ) is an area beyond and adjacent to the territorial sea and limited to 200 nautical miles (approximately 370.4 km) from the baseline where the coastal state has sovereign rights for the purpose of exploring and exploiting, conserving and managing the natural resources and with regard to other activities for the economic exploitation and exploration of the zone while the vessels of other states can freely navigate. It should be also mentioned that in the case of overlapping zones, the boundary is presumed to conform to the equidistance principle or it is explicitly described in a multilateral treaty.

Fig. 1 Arctic Ocean exclusive economic zone (Humpert 2012)

Thus, the territory of countries in the Arctic region is limited to the EEZs. The waters beyond the EEZs are considered the "high seas" or international waters which are not owned by any country (see Fig. 1).

According to Fig. 1 there are three high sea pockets in the Arctic region. The first pocket of around 250,000 km^2 which is called 'the Banana hole' or 'the Loop Sea' is located in international waters of the Norwegian Sea and is a gap between the EEZs of Norway, Iceland, the Faroe Islands and Greenland. The second pocket of around 175,000 km^2 which is called 'the Loop hole' is located in the Barents Sea and has been a source of tension between Norway and Russia for almost 40 years. In 2010 both countries signed a treaty concerning Maritime Delimitation and Cooperation in this region (Government of Norway 2010). The third pocket of around 2,800,000 km^2 is located at the north of the Arctic region and is not owned by any country.

Although the rules regulating the rights of countries in the Arctic region are clearly defined in the UNCLOS (1982), the current boundaries are not final yet. Additionally, the UNCLOS provides that "all coastal states have a continental shelf extending 200 nautical miles from coastal baselines or beyond 200 nautical miles if the shelf is a natural prolongation of its land territory". In that case the coastal states have sovereign rights over the natural resources of the seabed and subsoil of the continental shelf as well as jurisdiction over certain activities like marine scientific research. The outer limit of the continental shelf shall not exceed 350 nautical miles (approximately 648.2 km) from the baselines from which the breadth of the territorial sea is measured or shall not exceed 100 nautical miles from the 2500 m isobaths, which is a line connecting the depth of 2500 m. If the coastal state does not explore the continental shelf or exploit its natural resources, “no one may undertake these activities without the express consent of the coastal State” (UNCLOS 1982). The rights of the coastal country over the continental shelf do not affect the legal status of the superjacent waters or of the air space above them. The delimitation of the continental shelf between countries with opposite or adjacent coasts shall be effected by agreement “on the basis of international law in order to achieve an equitable solution” (UNCLOS 1982).

Unsettled demands concerning the demarcation of the continental shelves under the Arctic Ocean is potential source of conflict among different states. Currently, almost all Arctic states conduct scientific research to determine if its continental shelf extends beyond 200 nautical miles and, thus, have their claims on the Arctic which are considered by United Nations Commission on the Limits of the Continental Shelf. For instance, in 2001 Russia was the first to submit its claim to the United Nation (UN). The territory claimed by Russia is a large portion of the Arctic within its sector, extending to but not beyond the geographic North Pole. One of the arguments was a statement that Lomonosov Ridge, an underwater mountain ridge passing near the Pole, and Mendeleev Ridge on the Russian side of the Pole are extensions of the Eurasian continent. In 2002 the UN Commission sent the submission back for lack of evidence, recommending additional research. In 2015 Russia revised a new submission that included the Mendeleev elevation and in February 2016 it added the Chukchi high plain to the claim (United Nations 2017).

As for other countries, in 2006 Norway made an official submission and provided arguments to extend the Norwegian territory beyond the EEZ in the Western Nansen Basin in the Arctic Ocean, and the ‘Banana Hole’ in the Norwegian Sea (United Nations 2009). In 2014 Denmark that has the nearest coastline to the North Pole argued that the Lomonosov Ridge is in fact an extension of Greenland (United Nations 2014). The countries of Iceland and Denmark along with the Faroe Islands are also working on extending their continental shelf outside their EEZ in the ‘Banana Hole’ (Hund 2014). In 2016 Canada had announced that in 2018 it would file a claim which includes the North Pole (Sevunts 2016). As for the United States, since they had not ratified the UNCLOS, they are not eligible to stake an official claim to an extended continental shelf, however, obviously, they may also have a potential claim for the continental shelf north of Alaska. A series of maps with known claims and agreed boundaries, as well as potential areas that might be claimed in the future in the region are provided in IBRU: Centre for Borders Research - Durham University (2015).

Thus, there are many disputes among Arctic countries on how to determine who has the right to Arctic resources. The interest in the Arctic was also observed by many non-Arctic countries such as China, Japan, Republic of Korea, India, France,

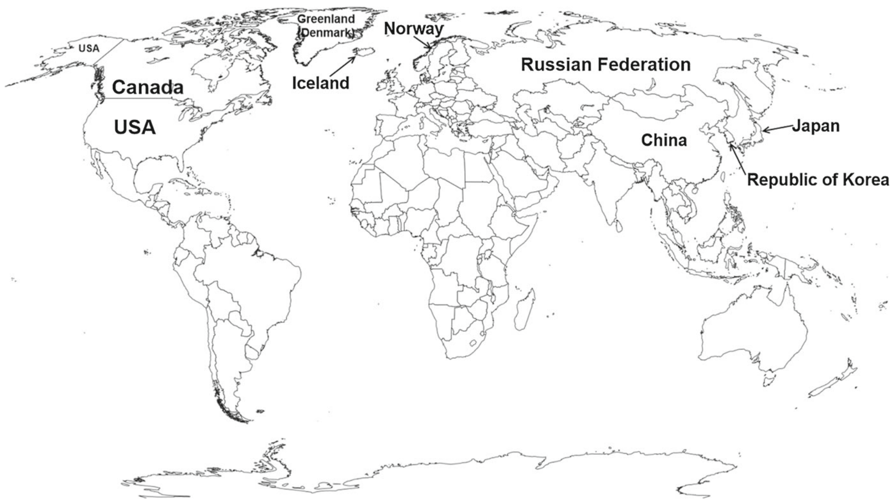

Fig. 2 Arctic and non-Arctic states

Germany, the Netherlands, Poland, Spain, the United Kingdom, etc. In other words, we can observe an intersection of mutual interests concerning the Arctic region.

3 Methodology

3.1 Problem Statement

Consider a set of objects X characterized by a set of parameters K and a set of agents N which are interested in these objects. The problem lies in the evaluation of the utility of an object for each agent and the intensity of mutual interests' intersection.

More precisely, since we consider the Arctic Region, a set of objects X is a set of areas in the north of the Arctic Circle and a set of agents N is a set of countries with some interest in this region. A list of countries we are focused on and their position in the world map are presented on Fig. 2.

Arctic States include eight countries five of which have a direct access to the Arctic Ocean. We focus on the United States, Russian Federation (Russia), Canada, Denmark (including Greenland and the Faroe Islands), Norway, Iceland and their EEZs capture in Arctic waters. Non-Arctic countries under consideration are China, Japan and the Republic of Korea (South Korea). The total number of interested parties is nine, i.e., $|N| = 9$.

We focus on the areas in Arctic region which are located to the north of 63° North latitude. The region is divided into 640,000 areas, i.e., $|X| = 640{,}000$ (see Fig. 3). Each area covers a territory of approximately 50 km^2.

Each area $x \in X$ is located at some distance from each country and possesses some natural resources. We consider four main natural resources: oil (O), gas (G), fish (F) and maritime (M) resources. Information on availability of each resource was

Fig. 3 Arctic region splitting (800 × 800 areas)

taken from Gautier et al. (2009) and Aleskerov and Victorova (2015). The graphical representation of that information is presented in Aleskerov and Shvydun (2017).

Among 640,000 areas we consider only areas that do not belong to EEZ of any country. The total number of areas is 59,121. If we exclude areas that do not contain natural resources, the total number will decrease to about 9281 areas.

Thus, we can evaluate the level of interest of each country in each area.

3.2 Utility Functions

Denote by $f(O, x)$, $f(G, x)$, $f(F, x)$, $f(M, x)$ the volume of oil, gas, fish and shipping routes in area $x \in X$, and by $u_k^O(x)$, $u_k^G(x)$, $u_k^F(x)$, $u_k^M(x)$ the utility of natural resources in the area for country $k \in N$. In Gautier et al. (2009) there are five gradations of oil and gas volume which we transformed into a 0–4 scale. Information about other resources is provided in a binary scale in Aleskerov and Victorova (2015).[1]

Assume that the interest of a country in each resource is constant in its exclusive economic zone (EEZ), proportional to the distance to the area outside its EEZ ($d^{EEZ} \approx$ 370.4 km), and equal to zero after some distance d^*. Then we can evaluate the level

[1] More detailed study can be performed on this issue.

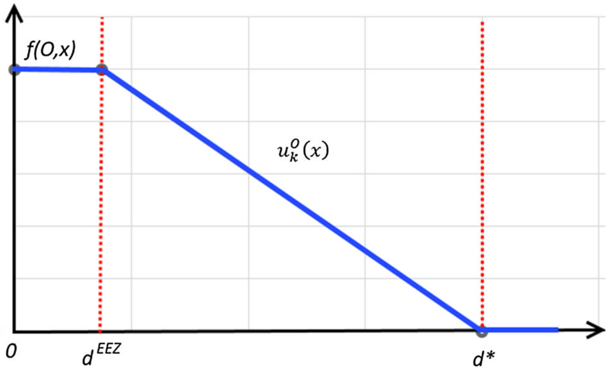

Fig. 4 Utility function $u_k^O(x)$

of interest of each country in natural resources located in some area by the following formulae.

- Natural resources

$$u_k^{Res}(x) = \begin{cases} f(Res, x), & if \quad d_k(x) \leq d^{EEZ}, \\ f(Res, x) \cdot \left(\frac{d^* - d_k(x)}{d^* - d^{EEZ}}\right), & if \quad d^{EEZ} < d_k(x) < d^*, \\ 0, & if \quad d_k(x) \geq d^*, \end{cases} \tag{1}$$

where Res is one of natural resources ($Res \in \{O, G, F\}$), $d_k(x)$ is the distance from the closest point of country $k \in N$ to area $x \in X$. A graphical representation of oil utility function $u_k^O(x)$ is provided on Fig. 4.

- Maritime routes

Contrary to the utility of oil, gas and fish, the interest in shipping routes should be calculated differently since it is less dependent on the distance. Moreover, the interest of each country in shipping routes of Arctic region actually depends on the usage of maritime routes. Thus, based on transit statistic for the last 2 years provided by the Northern Sea Route Information Office (The CHNL Information Office 2016) we calculated the total gross register tonnage[2] (GRT) of all vessels that sailed from or to a particular country and used this measure to evaluate the importance of the shipping routes for each country (Aleskerov and Shvydun 2017).

Let Imp_k be the importance for country k of shipping routes in Arctic region. Then we can assume that the utility of shipping routes of country $k \in N$ in area $x \in X$ is calculated by the following formula

$$u_k^M(x) = f(M, x) \cdot \max\left(Imp_k, \left(\frac{d^* - d_k(x)}{d^* - d^{EEZ}}\right)\right). \tag{2}$$

The formula has the following interpretation. Areas, which are very close to a country, have higher utility than the utility of more distant areas. However, the interest

[2] Gross register tonnage or gross tonnage (GT) represents the total internal volume of cargo vessels (World Ocean Review 2016).

in distant areas cannot be less than the importance of shipping routes in Arctic region of a particular country.

- Total utility evaluation

Since an exclusive economic zone (EEZ) of a country gives it exclusive rights to resources such as oil and gas, the interest of other countries in natural resources in that area is assumed to be zero. Moreover, if an area is in the EEZ of a country, it is not taken into account in the allocation procedure discussed in this Section. We also consider only those areas that do not belong to the territory of any country in the Arctic region.

The total utility of each zone in the Arctic can be evaluated by different techniques. For instance, it can be aggregated into a single value using methods presented in Aizerman and Aleskerov (1995), Aleskerov (1985) and Aleskerov and Subochev (2013). In this paper we calculate the total utility of each area $u_k^T(x)$ as

$$u_k^T(x) = \alpha_O \cdot u_k^O(x) + \alpha_G \cdot u_k^G(x) + \alpha_F \cdot u_k^F(x) + \alpha_M \cdot u_k^M(x), \tag{3}$$

where parameters $\alpha_O, \alpha_G, \alpha_F, \alpha_M$ correspond to the importance of oil, gas, fishing and maritime resources for interested countries. It should be mentioned here, that generally each country might evaluate natural resources differently, based on its industrial base, needs of the citizens, etc. However, for now, we assume that each country evaluates each resource equally.

It is necessary to note that since oil, gas and fishing resources require maritime access to the areas, we did not consider shipping routes as an additional resource of our model ($\alpha_M = 0$). For simplicity we also consider that all the other natural resources are equally valuable for all the countries ($\alpha_O = \alpha_G = \alpha_F = 1$). The analysis of preferences over natural resources requires a further consideration and is a topic of future research. However, we should note the presented models can be easily extended to the case of unequal importance of different resources.

As for the choice of the threshold level d^*, we assume the level being equal to 4000 km ($d^* = 4000$) which means that Arctic states are interested in the whole region while non-Arctic states are interested only in some parts of it. We have also checked the threshold level equal to 3000 km and 5000 km and obtained similar results of areas allocation.

Thus, we can evaluate an interest of each country in a specific area of Arctic region and find areas of the most interest. The areas can also be ranked lexicographically or by some other procedures according to the preferences of each country.

3.3 Goal Functions

To resolve the intersection of mutual interest among different countries it is necessary to propose some allocation of areas fair in some sense for each country. The fairness of the allocation can be evaluated differently; in our paper it is based on the satisfaction level of each country $S_k(P)$ which is calculated as

$$S_k(P) = \sum_{x \in X:(x,k) \in P} \left(u_k^T(x)\right) - \sum_{x \in X:(x,k) \notin P} \left(u_k^T(x)\right), \tag{4}$$

where P is a binary relation $P \subset X \times N$ that characterizes the final allocation of areas with the following constraints

(1) $\forall x \in X \exists k \in N : (x, k) \in P$;
(2) $\nexists k, l \in N, l \neq k : (x, k) \in P \& (x, l) \in P$.

In other words, each area should be allocated to some country and no areas can be allocated to more than one country.

Thus, the satisfaction level of a country is calculated as the difference between the total utility of areas allocated to this country, and the potential total utility of areas not allocated to the country. If the satisfaction level is equal for all countries, we assume that all countries are satisfied with the proposed allocation.

Additionally, we can check envy-freeness of the allocation. Let us remind that allocation of areas is envy-free if no agent has an incentive to exchange her allocated part with any other agent (Brams and Taylor 1996). We denote by $\tilde{S}_k(P)$ the envy-free satisfaction level of each country $k \in N$ and evaluate it as

$$\tilde{S}_k(P) = \sum_{x \in X:(x,k) \in P} \left(u_k^T(x)\right) - \max_{t \in Y \setminus \{k\}} \left(\sum_{x \in X:(x,t) \in P} \left(u_k^T(x)\right) \right). \tag{5}$$

The allocation is envy-free if $\forall k \in N \tilde{S}_k(P) \geq 0$.

In the next Section, we propose several models of the areas allocation and evaluate the satisfaction level for all countries under consideration.

4 Models of Single Areas Allocation in the Arctic

Next we consider several models of areas allocation fair in some sense for each country. First, we consider several allocations that do not take into account the satisfaction level of each country. The proposed models are based on the distance to the area and the total interest in them. Second, we propose several models that take into consideration the satisfaction level of the countries. These models are based on the idea that each country should have the same satisfaction level. In other words, these models propose a fair allocation of areas. The models are based on techniques which are used in fair division problems and in linear optimization models. Finally, we propose another model of areas allocation which are based on the idea of superposition.

Again, we consider areas that do not belong to the EEZ of any country. The models are applied on areas with natural resources while other areas are allocated with respect to the obtained allocation of areas and the distance to the countries.

To test models we consider various scenarios. The main goal of them is to define areas of mutual interest in the Arctic region, i.e., areas that are allocated to different countries by various scenarios.

Note that classical techniques which are used in assignment problem (Kuhn 1955) are not applicable to the problem since the goal of these models is focused on the maximization of the linear function (e.g. the total satisfaction level of all countries or the total utility of all countries from allocated areas). Thus, the results of these models are equal to the allocation of zones to the most interested country (see Sect. 4.1).

4.1 Areas Allocation Regardless the Satisfaction Level

Consider the simplest allocation model that does not take into account the satisfaction level of each country. Intuitively, an area should be allocated to the closest country (scenario 1) or to the country with the highest utility level according to our evaluations (scenario 2). Then the final allocation P should satisfy the following conditions

(1) $(x, k) \in P \Leftrightarrow \forall l \in N d_k(x) \leq d_l(x)$ (scenario 1);
(2) $(x, k) \in P \Leftrightarrow \forall l \in N\, u_k^T(x) \leq u_l^T(x)$ (scenario 2).

Note that if two or more countries have the same interest in a specific area or the same distance to it, the area will be allocated to the country which has the lowest satisfaction level.

More detailed information is provided in Aleskerov and Shvydun (2017). Obviously, using introduced model of utility values all areas in the Arctic are allocated among the Arctic states. Moreover, due to the form of the utility function which is based on the distance, scenarios 1 and 2 provided the same results of areas allocation. Thus, we can combine these scenarios and we present the satisfaction level of each country in Table 1.

According to Table 1, the total satisfaction level of most countries is negative for the threshold level $d^* = 4000$. It can be explained by the fact that countries are interested in most part of Arctic region and, obviously, none of them can obtain

Table 1 Satisfaction level and number of allocated areas with natural resources for scenarios 1, 2

Country	Scenarios 1, 2	
	Satisfaction level	Allocated areas
USA	– 6224	21
Russia	6161	7512
Canada	– 6016	790
Denmark	– 6819	96
Norway	– 4171	862
Iceland	– 2607	0
China	0	0
Japan	– 791	0
South Korea	– 10	0
Total	– 20,477	9281

the whole region. The satisfaction level is positive only for Russia. This positive total satisfaction level can be explained by long maritime borders of Russia and low number of close neighbors which are interested in its adjacent marine areas (contrary to, for instance, Canada). The satisfaction level is zero for China since for the threshold level $d^* = 4000$ this country is not interested in the region due to the distance. As for the number of allocated zones, no areas are allocated to non-Arctic countries. Russia, Denmark and Canada are the countries with the largest number of allocated zones of Arctic region.

Since the satisfaction level for other countries is quite different, scenarios 1, 2 are not fair according to our model. Consider also the envy-freeness of these scenarios (see Table 2) where the value in cell corresponds to the difference of utilities that country (in row) obtained by allocation procedure and that could obtain by exchange with another country (in column). Negative values mean that the allocation is not envy-free. Positive values mean that a country does not envy another country.

According to Table 2, scenarios 1, 2 of areas allocation are not envy-free. Thus, we conclude that these scenarios cannot be used as possible solutions of areas allocation problem since they provide different satisfaction level and do not guarantee envy-freeness.

4.2 Areas Allocation Based on the Satisfaction Level

Suppose there is an initial allocation l_1 of each area of the Arctic region to a particular country. Such allocation l_1 can be represented as a binary relation P_{l_1} which was defined in the previous Section. Then we can evaluate the satisfaction level of each country. If the satisfaction level of all countries is equal, the procedure of areas allocation terminates. Otherwise, we should construct another allocation which will be fairer in terms of the satisfaction level. This step is performed by the exchange procedure consisting in the re-assignment of an area from one country to another one. Based on the satisfaction levels we can define the most and the less satisfied country and exchange areas between them and denote them by k_1 and k_2, respectively. Then the exchange procedure is performed for an area $x \in X$ which satisfies the following conditions

$$\begin{aligned}
&(x, k_1) \in P_{l_1};\\
&u^T_{k_2}(x) \neq 0;\\
&\frac{u^T_{k_2}(x)}{u^T_{k_1}(x)} \rightarrow max.
\end{aligned}$$

If several areas satisfy this criterion, we choose the area which is closer to country k_2 than to country k_1. If there are no areas that country k_1 can give to country k_2, then we consider the next most satisfied country. Thus, we obtain the next distribution l_2.

The criterion for the choice of exchanging area $x \in X$ between countries is similar to the criterion used for adjusted winner procedure which is designed for the division of goods among two agents (Brams and Taylor 1996). First, area $x \in X$ should belong to the most satisfied country. Second, area $x \in X$ should be valuable for unsatisfied

Table 2 Envy-freeness for scenarios 1, 2

Country in row envies country in column	USA	Russia	Canada	Denmark	Norway	Iceland	China	Japan	South Korea	Envy-freeness
USA	0	−5825	−343	−1	28	28	28	28	28	−5825
Russia	7805	0	7287	7754	6795	7827	7827	7827	7827	0
Canada	573	−5268	0	529	−49	600	600	600	600	−5268
Denmark	67	−5161	−510	0	−962	85	85	85	85	−5161
Norway	1209	−3560	701	1138	0	1220	1220	1220	1220	−3560
Iceland	−4	−1142	−274	−58	−1130	0	0	0	0	−1142
China	0	0	0	0	0	0	0	0	0	0
Japan	−6	−784	0	0	0	0	0	0	0	−784
South Korea	0	−10	0	0	0	0	0	0	0	−10

country. Finally, the exchange should be performed for the area which is valuable as much as possible for unsatisfied country and as less as possible for satisfied country.

Thus, some new allocation is obtained and the whole procedure repeats again. There are different criteria that can be used to terminate the exchange procedure. Some examples of them is provided below.

(1) Maximization of the total satisfaction level. The procedure terminates at step i if

$$\exists i : \forall j < i \sum_k S_k(P_i) \geq \sum_k S_k\left(P_j\right) \& \sum_k S_k(P_i) > \sum_k S_k(P_{i+1});$$

(2) Minimization of the difference between satisfaction levels of the most satisfied and unsatisfied countries. The procedure terminates at step i if

$$\exists i : \forall j < i \max_k S_k(P_i) - \min_k S_k(P_i) \leq \max_k S_k\left(P_j\right) - \min_k S_k\left(P_j\right) \&$$
$$\max_k S_k(P_i) - \min_k S_k(P_i) < \max_k S_k(P_{i+1}) - \min_k S_k(P_{i+1});$$

(3) Maximization of the satisfaction level of the most unsatisfied country. The procedure terminates at step i if

$$\exists i : \forall j < i \min_k S_k(P_i) \geq \min_k S_k\left(P_j\right) \& \min_k S_k(P_i) > \min_k S_k(P_{i+1}).$$

Obviously, if the first criterion is applied to terminate the procedure, the allocation of zones should be performed to the most interested countries. If two or more countries have the same interest in a specific area, the exchange procedure will allocate the area to the country which has the lowest satisfaction level. This allocation has been already considered in the previous Section and it was shown that it does not guarantee envy-freeness or fairness.

We can also use the second criterion to stop the exchange procedure. However, this criterion is not aimed at increasing the total satisfaction level of all countries which is also an important criterion of efficiency of the proposed allocation. In other words, there can be several areas allocations with the same difference between satisfaction levels of the most satisfied and unsatisfied countries, and it is not guaranteed that the exchange procedure provides the best allocation in terms of the total satisfaction level. Thus, we will not consider the second criterion.

In this Section we are focused on the third criterion. The maximization of the satisfaction level of the most unsatisfied country consists in the search of the most unsatisfied country and the re-assignment to that country of the area that was primarily allocated to the other country with higher satisfaction level.

The results of this model are not that obvious since they depend on the initial allocation of areas. We propose different initial allocations of areas and evaluate them in terms of the total satisfaction level and envy-freeness. The list of proposed allocations is presented in Table 3.

Thus, we consider 12 scenarios of initial areas allocation and, by performing the exchange procedure, obtain different final allocations. Again, we do not consider those

Table 3 List of initial allocations

Name	Initial allocation of zones in the Arctic
Scenario 3	All areas are allocated to the country closest to that area
Scenario 4	All areas are allocated randomly
Scenario 5	All areas are allocated to the most interested countries in terms of the total utility
Scenario 6	All areas are allocated to the USA
Scenario 7	All areas are allocated to Russia
Scenario 8	All areas are allocated to Canada
Scenario 9	All areas are allocated to Denmark
Scenario 10	All areas are allocated to Norway
Scenario 11	All areas are allocated to Iceland
Scenario 12	All areas are allocated to China
Scenario 13	All areas are allocated to Japan
Scenario 14	All areas are allocated to the Republic of Korea

areas that belong to EEZ of countries under consideration. More detailed information is provided in Aleskerov and Shvydun (2017).

In Table 4 we present the satisfaction level of each country according to different scenarios. To obtain final allocation the model performed from 3,000,000 to 8,000,000 exchange iterations.

Since all countries are interested in most part of Arctic region, their satisfaction levels are negative. Non-Arctic countries are the most satisfied states by each scenario. It can be explained by the fact that their interests and total number of areas in Arctic are lower than for Arctic states. As for Arctic states (except Iceland), their satisfaction levels are almost equal according to different scenarios. If we consider the total satisfaction level of allocations, scenario 13 provides better results, however, the level of improvement is insignificant. We can also observe that these allocations are almost envy-free (Aleskerov and Shvydun 2017). Comparing to scenarios 1, 2 we can conclude that although the total satisfaction level of each allocation is lower, allocations by scenarios 3–14 are more fair for all countries.

Additionally, we provide information about the number of allocated zones according to scenarios 3–14 (see Table 5).

If we look to the total number of allocated areas, it is seen that no areas are allocated to non-Arctic countries that can be explained by the following reasons. Non-Arctic countries are very distant from the Arctic region, hence their interest in any area is lower in comparison to Arctic states. Moreover, if the threshold level $d^* = 4000$ is used the total dissatisfaction level of non-Arctic countries from not receiving any territory in Arctic region is less than the dissatisfaction level of Arctic countries. As for the other states, we can observe that scenarios 3–14 allocate almost the same number of areas to each country. Russia, Denmark and Canada got the largest number of areas with natural resources (around 2000 areas each). Interestingly, Iceland did not receive any area. It can be explained by the fact that, according to our evaluations, the total interest of Iceland in Arctic is much less that the total interest of other Arctic states.

Table 4 Satisfaction level of countries for scenarios 3–14

# of scenario	Countries									
	USA	Russia	Canada	Denmark	Norway	Iceland	China	Japan	South Korea	Total
Scenario 3	– 3423.7	– 3423.8	– 3423.7	– 3423.7	– 3423.8	– 2607.3	0	– 790.8	– 9.6	– 20,526.4
Scenario 4	– 3423.6	– 3423.8	– 3423.8	– 3423.8	– 3423.7	– 2607.3	0	– 790.8	– 9.6	– 20,526.4
Scenario 5	– 3423.7	– 3423.8	– 3423.7	– 3423.7	– 3423.8	– 2607.3	0	– 790.8	– 9.6	– 20,526.4
Scenario 6	– 3423.8	– 3423.7	– 3423.8	– 3423.6	– 3423.5	– 2607.3	0	– 790.8	– 9.6	– 20,526.1
Scenario 7	– 3423.9	– 3423.9	– 3423.9	– 3423.8	– 3423.9	– 2607.3	0	– 790.8	– 9.6	– 20,527.1
Scenario 8	– 3423.7	– 3423.7	– 3423.8	– 3423.7	– 3423.6	– 2607.3	0	– 790.8	– 9.6	– 20,526.2
Scenario 9	– 3423.4	– 3423.4	– 3423.2	– 3423.4	– 3423.3	– 2607.3	0	– 790.8	– 9.6	– 20,524.4
Scenario 10	– 3423.4	– 3423.5	– 3423.4	– 3423.4	– 3423.5	– 2607.3	0	– 790.8	– 9.6	– 20,524.9
Scenario 11	– 3423.7	– 3423.8	– 3423.9	– 3423.9	– 3423.9	– 2607.3	0	– 790.8	– 9.6	– 20,526.9
Scenario 12	– 3423.8	– 3423.8	– 3423.8	– 3423.8	– 3423.9	– 2607.3	0	– 790.8	– 9.6	– 20,526.8
Scenario 13	– 3422.8	– 3422.7	– 3422.7	– 3422.7	– 3422.8	– 2607.3	0	– 790.8	– 9.6	– 20,521.4
Scenario 14	– 3423.7	– 3423.7	– 3423.8	– 3423.8	– 3423.6	– 2607.3	0	– 790.8	– 9.6	– 20,526.3

Table 5 Number of allocated zones with natural resources for scenarios 3–14

Country	# of scenario											
	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9	Scenario 10	Scenario 11	Scenario 12	Scenario 13	Scenario 14
USA	1515	1513	1515	1514	1514	1514	1514	1515	1515	1514	1518	1514
Russia	2102	2107	2102	2112	2112	2114	2098	2110	2109	2109	2084	2114
Canada	1960	1950	1960	1947	1953	1947	1945	1945	1950	1950	1931	1947
Denmark	2443	2451	2443	2449	2452	2452	2463	2450	2450	2454	2484	2452
Norway	1261	1260	1261	1259	1250	1254	1261	1261	1257	1254	1264	1254
Iceland	0	0	0	0	0	0	0	0	0	0	0	0
China	0	0	0	0	0	0	0	0	0	0	0	0
Japan	0	0	0	0	0	0	0	0	0	0	0	0
South Korea	0	0	0	0	0	0	0	0	0	0	0	0
Total	9281	9281	9281	9281	9281	9281	9281	9281	9281	9281	9281	9281

Table 6 Satisfaction level and the number of allocated areas for combined scenario

Country	Combined scenario	
	Satisfaction level	Allocated areas
USA	– 3425.2	1512
Russia	– 3428.7	2110
Canada	– 3428.3	1948
Denmark	– 3412.7	2458
Norway	– 3429.2	1253
Iceland	– 2607.3	0
China	0	0
Japan	– 790.8	0
South Korea	– 9.6	0
Total	– 20,531.8	9281

Since the total number of allocated areas to each country is almost equal by scenarios 3–14, we analyzed information about allocated areas in order to understand how different these scenarios are. It was observed that 90.5% of areas was allocated to the same country (8401 of 9281 areas) by scenarios 3–14. About 9.2% of areas was distributed among two countries (849 of 9281 areas), however, at least 9 scenarios give 682 of these 849 areas to the same country. The remaining 0.3% of areas was allocated between three or more states (31 of 9281 areas) where each zone covers a territory of approximately 50 km^2. We may consider these areas as zones that have the most disputes. Nevertheless, we may conclude that scenarios 3–14 are similar as allocated areas and satisfaction levels of interested countries are almost equal.

Additionally, we combine scenarios 3–14 since they provide similar areas distribution. There can be used different techniques for scenarios aggregation, for instance, we can apply the same exchange procedure for areas which are not allocated to a single country. In our work we used the majority rule for areas which are not allocated to the same country (Aizerman and Aleskerov 1995). The results for the combined scenario are provided in Table 6.

We should note that the combined scenario does not guarantee that satisfaction levels of Arctic states will be equal. Indeed, according to Table 6, Denmark is more satisfied than any other country with allocated areas. Moreover, the total satisfaction level for this scenario is lower than for the scenarios from Table 4. However, we consider this scenario as more consistent since all areas are allocated by majority with respect to scenarios 3–14.

4.3 Areas Allocation Based on Superposition Model

4.3.1 Superposition Allocation Model to the Most Interested Country

To allocate Arctic zones among interested countries we also use another models based on the idea of superposition (see Aizerman and Aleskerov 1995). In general, superpo-

Table 7 Sequence of countries for scenarios

No.	Sequence of countries
Scenario 15	Norway → Russia → Denmark → Japan → USA → Canada → Republic of Korea → China → Iceland
Scenario 16	Japan → Republic of Korea → Denmark → USA → Iceland → China → Canada → Norway → Russia
Scenario 17	USA → Canada → Denmark → Iceland → Republic of Korea → Russia → China → Japan → Norway
Scenario 18	Denmark → Canada → China → Republic of Korea → Japan → USA → Norway → Iceland → Russia

sition consists in sequential application (composition) of different functions such that the output of the previous step is the input for the next step. In our case as a function we add a new party and change the allocation of zones at each step.

In Shvydun and Aleskerov (2017) we proposed a superposition model which sequentially added interested countries into the allocation procedure while the exchange of zones between two countries is performed by a simple majority rule. In other words, a zone of mutual interests is allocated to country *B* if the total number of resources in which country *B* is interested more than country *A* is more than or equal to $50\% + 1$ of the total number of resources in this zone.[3] The advantage of the proposed model is in its low computational complexity, however, it does not provide the equity of satisfaction levels for interested countries. In other words, the model does not take into account the total satisfaction level of each country since it is based on the idea that any area should be allocated to the most interested country.

To allocate Arctic zones among interested countries we propose another model. Suppose there is only one country which is interested in the Arctic region. Therefore, the whole region will be entirely allocated to this country. Next, let us add another interested in the region country. To reduce the dissatisfaction of the second country, the first country can transfer some zones which are more valuable to the second country. To exchange zones we can use the criterion from Sect. 4.2. When the exchange procedure terminates, we can add one more country which is interested in the region. Similarly, we can allocate to a new country some zones of the Arctic. Thus, we can add all interested countries and allocate zones among them. The main intuition of the proposed model is that it may perform less exchange iterations since we consider fewer countries on previous steps.

The superposition operation is not commutative, i.e., the change of the order may lead to completely different results. The properties of some multi-criteria choice procedures are provided in Aizerman and Aleskerov (1995) and Shvydun (2015). Thus, we should consider various sequences of interested parties. To test the model we randomly selected some sequences of countries which are presented in Table 7.

[3] Note that in the case when single criterion (for instance, the total utility) is used for the exchange procedure, the described model is similar to simple maximization model, so the change of the sequence of countries does not change the final results, i.e., the final allocation is equal to scenarios 1, 2.

In Table 8 we provide information about the satisfaction level of each country according to different scenarios. Additionally, we present information about the number of allocated zones according to scenarios 15–18 (see Table 9). More detailed information is provided in Aleskerov and Shvydun (2017).

As it is shown in Table 8, satisfaction levels are almost equal for all Arctic states (except Iceland) with a threshold level $d^* = 4000$. Non-Arctic countries are again the most satisfied states by each scenario. If we consider the total satisfaction level of allocations, scenario 18 provides slightly better results. The proposed scenarios of areas allocation are almost envy-free (Aleskerov and Shvydun 2017). As for the analysis of allocated zones, 94% of areas are distributed to the same country (8731 of 9281) while the remaining 6% of areas (550 of 9281) are allocated among two or three countries. Thus, the proposed scenarios are almost similar and can be combined as we did it in Sect. 4.2.

We should also note that the obtained results are similar to scenarios 3–14. About 88.3% of areas are allocated to the same country by scenarios 3–14 and scenarios 15–18. As for the complexity of the superposition model, the total number of exchange iterations varies from 1,000,000 (scenario 17) to 8,000,000 (scenario 18). The number of operations is similar to the model from Sect. 4.2, however, the minimal number of required iterations is lower for superposition model.

4.3.2 Areas Allocation Based on the Adjusted Winner Procedure

Consider another model of areas allocation. Among classical fair division models, there is a well-known adjusted winner procedure (Brams and Taylor 1996) which ensures that the final allocation is proportional (each side receives a piece that she perceives to be at least $1/n$ of the whole), envy-free (no agent has incentives to exchange her allocated part of the object with any other agent) and Pareto-optimal (no other allocation can make one party better off without making the other party worse off). However, the adjusted winner procedure is designed for the case of just two parties, thus, it cannot be applied in our case. In Brams et al. (2017) there were also proposed two envy-free and Pareto-optimal algorithms for the case of two agents and linear order of their preferences over even number of indivisible items which is also not the case in our problem since we consider more than two agents with no strict ranking over the areas.

In Sect. 4.2 we proposed a model which uses the criterion of zones exchange which is similar to the adjusted winner procedure. In this Section we propose a model based on the idea of superposition which is quite similar to the adjusted winner procedure.

The idea of the model is the following. Suppose all countries are somehow divided into two different groups. Then, if we evaluate the group interest for all areas in the Arctic and rank them, it is possible to allocate all zones between these two groups using standard adjusted winner techniques. After the procedure applied, we consider each group individually and somehow split countries included in the group into two different subgroups. Then we can allocate all areas of the group among two subgroups using the same allocation procedure. Thus, we can apply the adjusted winner procedure

Table 8 Satisfaction level of countries for scenarios 15–18

# of scenario	Countries									
	USA	Russia	Canada	Denmark	Norway	Iceland	China	Japan	South Korea	Total
Scenario 15	− 3423.4	− 3427.4	− 3423.8	− 3423.0	− 3422.2	− 2607.3	0	− 790.8	−9.6	−20,527.5
Scenario 16	− 3420.8	− 3423.6	− 3423.6	− 3428.3	− 3423.8	− 2607.3	0	− 790.8	−9.6	−20,527.8
Scenario 17	− 3424.1	− 3424.0	− 3423.7	− 3423.7	− 3424.0	− 2607.3	0	− 790.8	−9.6	−20,527.2
Scenario 18	− 3422.8	− 3423.5	− 3423.3	− 3421.5	− 3423.6	− 2607.3	0	− 790.8	−9.6	−20,522.4

Table 9 Number of allocated zones with natural resources for scenarios 15–18

Country	# of scenario			
	Scenario 15	Scenario 16	Scenario 17	Scenario 18
USA	1510	1518	1515	1513
Russia	2106	2104	2117	2091
Canada	1950	1943	1949	1939
Denmark	2453	2459	2452	2478
Norway	1262	1257	1248	1260
Iceland	0	0	0	0
China	0	0	0	0
Japan	0	0	0	0
South Korea	0	0	0	0
TOTAL	9281	9281	9281	9281

until each group contains only one member. The obtained allocation of zones is the final allocation.

Formally, the proposed model can be divided into four main stages:

(1) Divide a group of countries into two equal subgroups;
(2) Evaluate the interest to available areas for each subgroup;
(3) Apply the adjusted winner procedure and allocate all available areas among two subgroups;
(4) For each subgroup apply steps 1–3 if it contains several countries.

There are several questions left for this model. First, how does the procedure split a set of countries into two equal subgroups. The second question is how to evaluate the group interest of areas in the Arctic region since each member of the group has different preferences.

The division of countries into two groups can be performed differently. For instance, all countries can be divided according to their geographical location, international relations, mutual interests, etc. However, the obtained division may be subjective and arguable, moreover, it would be difficult to find it, especially if some new countries interested in the Arctic emerge. Thus, we consider the division problem from the mathematical point of view and propose to divide all countries with respect to their level of interest in Arctic zones. In other words, on the first stage the division of the set of countries N into two subsets $N' \subset N$ and $N \backslash N'$ is performed by solving the following equation

$$\left| \sum_{x \in X} \sum_{k \in N'} \left(u_k^T(x) \right) - \sum_{x \in X} \sum_{k \in N \backslash N'} \left(u_k^T(x) \right) \right| \to min \qquad (6)$$

while on next steps when there is an intermediate allocation P among different groups of countries the division procedure of group N' into two subgroups N'' and $N' \backslash N''$ is performed as

$$\left| \sum_{k \in N''} \left(\sum_{\substack{x \in X: \\ (x, N') \in P}} u_k^T(x) - \sum_{\substack{x \in X: \\ (x, N') \notin P}} u_k^T(x) \right) - \sum_{k \in N' \backslash N''} \left(\sum_{\substack{x \in X: \\ (x, N') \in P}} u_k^T(x) - \sum_{\substack{x \in X: \\ (x, N') \notin P}} u_k^T(x) \right) \right| \to min. \qquad (7)$$

The interpretation of formula (6) is the following. On the first step none of areas are allocated to any group. Thus, the main idea is to construct two groups with almost the same total level of interest in the whole region. Formula (7) is an extended version of the formula (6). Since there are particular areas that were allocated to group N' (group $N \backslash N'$), countries from this group cannot obtain areas from another group $N \backslash N'$ (group N' respectively). Thus, we should take into account the total level of interest in areas that countries lost because they belong to a fixed group.

The next question is how to evaluate the group interest to each area. Since we consider a single allocation of areas, there is only one country from group N' which may receive some area $x \in X$. Thus, we calculate the group interest $u_{N'}^T(x)$ in areas as

$$u_{N'}^T(x) = 2 \cdot \max_{k \in N'} u_k^T(x) - \sum_{k \in N'} u_k^T(x). \qquad (8)$$

According to formula (8) we calculate the group interest as the difference between the total utility of the most interested country from group N' and the total interest of other countries. It should be noted here that the summation of the utilities does not always represent the interest of the group since the non-compensatory condition is not satisfied. In other words, it is possible that an area which is highly appreciated by one country and is not important for other countries will have a higher rank than areas which all countries are interested in.

Thus, we define how to split countries in subgroups and how to evaluate their interest. If we apply the adjusted winner procedure to allocate areas among two subgroups, we will obtain the allocation that satisfies both groups. The procedure continues for each subgroup until all subgroups contain only one country.

In Table 10 we provide information about the groups of countries and their division into subgroups according to the model.

Table 10 Groups of countries and their division into subgroups

<table>
<tr><th>Step 0</th><th>Step 1</th><th>Step 2</th><th>Step 3</th><th>Step 4</th><th>Step 5</th></tr>
<tr><td rowspan="9">China, Republic of Korea, Iceland, Russia, Japan, Denmark, Norway, USA, Canada</td><td rowspan="6">China, Republic of Korea, Iceland, Russia, Japan, Denmark</td><td rowspan="4">China, Republic of Korea, Iceland, Russia</td><td rowspan="3">China, Republic of Korea, Iceland</td><td rowspan="2">China, Republic of Korea</td><td>China</td></tr>
<tr><td>Republic of Korea</td></tr>
<tr><td colspan="2">Iceland</td></tr>
<tr><td colspan="3">Russia</td></tr>
<tr><td rowspan="2">Japan, Denmark</td><td colspan="3">Japan</td></tr>
<tr><td colspan="3">Denmark</td></tr>
<tr><td rowspan="3">Norway, USA, Canada</td><td rowspan="2">Norway, USA</td><td colspan="3">Norway</td></tr>
<tr><td colspan="3">USA</td></tr>
<tr><td>Canada</td><td colspan="3">Canada</td></tr>
</table>

Table 11 Satisfaction level and the number of allocated areas for scenario 19

Country	Scenario 19	
	Satisfaction level	Allocated areas
USA	– 2589.4	2051
Russia	– 2664.8	2298
Canada	– 4881.7	1126
Denmark	– 4471.2	1772
Norway	– 2589.3	2034
Iceland	– 2607.3	0
China	0	0
Japan	– 790.8	0
South Korea	– 9.6	0
Total	– 20,604.1	9281

In Tables 11 and 12 we present the satisfaction level and envy-freeness of each country. More detailed information is provided in Aleskerov and Shvydun (2017).

According to Table 11, the total satisfaction level of scenario 19 is similar to allocations proposed in Sect. 4.2. The USA, Russia and Norway are countries with the largest number of allocated areas while Iceland and non-Arctic states did not get any area. As for the satisfaction level of countries, we can observe that it is almost the same for the USA, Russia, Norway and Iceland. Canada and Denmark have almost the same satisfaction level as well. Comparing to scenarios 3–18, although individual satisfaction levels of countries with allocated areas are not that similar, it is almost equal in terms of the group interest.

Consider also the envy-freeness of scenario 19 (see Table 12). According to the proposed allocation, Canada, Denmark, Iceland and Japan are not envy-free. These countries would like to exchange their allocated areas with countries, for instance, like Russia and Norway. However, we can observe that the level of envy-freeness is lower than for scenarios that do not take into account the satisfaction level of each country.

Table 12 Envy-freeness for scenario 19

Country in row envies country in column	USA	Russia	Canada	Denmark	Norway	Iceland	China	Japan	South Korea	Envy-freeness
USA	0	−29	1298	614	1044	1839	1839	1839	1839	−29
Russia	850	0	2102	1190	1845	2880	2880	2880	2880	0
Canada	−763	−1104	0	−489	−67	820	820	820	820	−1104
Denmark	93	−586	446	0	−669	1255	1255	1255	1255	−669
Norway	1094	211	1297	841	0	2011	2011	2011	2011	0
Iceland	−11	−456	−354	−371	−1416	0	0	0	0	−1416
China	0	0	0	0	0	0	0	0	0	0
Japan	−524	−148	0	−29	− −90	0	0	0	0	−524
South Korea	−10	0	0	0	0	0	0	0	0	−10

4.4 Areas Allocation Based on Linear Optimization Model

Now we consider the problem of areas allocation as an integer linear programming problem. Let $x_{ij} = \{0, 1\}$ be the value showing if area j is allocated to country i, i.e., $x_{ij} = 1$ if it is true and $x_{ij} = 0$ otherwise. Since we study a single allocation of zones, we add the following constraint

$$\sum_i x_{ij} = 1 \quad for\ any \quad j \in X. \tag{9}$$

The satisfaction level of a country i can be defined as

$$\sum_j \left(-1 + 2 \cdot x_{ij}\right) \cdot u_i^T(j). \tag{10}$$

The goal is to find values x_{ij} that maximizes the satisfaction level of all countries, i.e.,

$$\sum_i \sum_j \left(-1 + 2 \cdot x_{ij}\right) \cdot u_i^T(j) \to max,$$

subject to (9) and the constraint that the satisfaction level of each pair of countries should be equal

$$\left| \sum_j \left(-1 + 2 \cdot x_{i_1 j}\right) \cdot u_{i_1}^T(j) - \sum_j \left(-1 + 2 \cdot x_{i_2 j}\right) \cdot u_{i_2}^T(j) \right| \leq p \quad \forall i_1, i_2 \in N$$

where p is a parameter that shows to which extent the satisfaction level of two countries can be different so the allocation of zones will still be fair. Ideally, $p = 0$.

A solution of this integer linear programming problem gives an allocation of zones among countries. In Table 13 we present the satisfaction level of each country.

According to Table 13, no areas were allocated to Iceland and non-Arctic countries again. The total satisfaction level of the USA, Russia, Canada, Denmark and Norway is similar. Although the satisfaction level of Russia and the USA is slightly higher than for Canada, Denmark and Norway, the proposed allocation provide worse results in terms of total satisfaction level than previous scenarios. We can also note that scenario 20 is similar to scenarios from Sects. 4.2 and 4.3.1. In terms of the total satisfaction level, scenario 20 provides better results, however, scenarios 3–18 are more fair in terms of individual satisfaction levels.

Consider also the envy-freeness of scenario 20 (see Table 14). According to the proposed allocation, the scenario 20 is not envy-free.

Table 13 Satisfaction level and number of allocated areas with natural resources for scenario 20

Country	Scenario 20	
	Satisfaction level	Allocated areas
USA	− 3374.1	1544
Russia	− 3376.3	2103
Canada	− 3447.8	1914
Denmark	− 3447.5	2468
Norway	− 3448.2	1252
Iceland	− 2607.2	0
China	0	0
Japan	− 790.8	0
South Korea	− 9.6	0
Total	− 20,501.5	9281

5 Shared Areas Allocation in the Arctic

Consider now a possibility of joint ownership of a same area. Again, we have some initial allocation of areas in Arctic region and the exchange procedure which is performed similarly to Sect. 4.2. However, joint ownership between two countries is allowed now. Such allocation l can be represented as a binary relation $P_l^{shared} \subset X \times N$ with the following constraint

$$\forall x \in X \exists k \in N : (x, k) \in P_l^{shared}.$$

Assume that the utility of joint ownership $u_k^{T,shared}(x)$ of area $x \in X$ for each country $k \in N$ is proportional to the total utility of countries that own the area, i.e.,

$$u_k^{T,shared}(x) = \frac{u_k^T(x)}{\sum_{y \in N:(x,y) \in P_l^{shared}} u_y^T(x)}.$$

The criterion for the joint ownership is the following: if performing the exchange procedure the same area is swapped more than m times between same two countries then this area is shared by them. To test the idea we limited the number of swaps to $m = 6$. Additionally, we restricted the total number of countries that can share the same area to two, i.e., $1 \leq \left|k \in N : (x, k) \in P_l^{shared}\right| \leq 2$. However, this parameter can be extended even up to the total number of countries involved.

Thus, we can define areas of joint ownership in the Arctic region. Below we consider the initial allocation of areas with respect to the distance to these areas. In other words, each area is allocated to the country closest to that area.

The results of the model application are provided in Aleskerov and Shvydun (2017). Let us evaluate scenario 21 in terms of the total satisfaction level (see Table 15).

According to Table 15, shared allocation provides similar results to a single allocation of areas with respect to the total satisfaction level. The USA, Russia, Canada,

Table 14 Envy-freeness for scenario 20

Country in row envies country in column	USA	Russia	Canada	Denmark	Norway	Iceland	China	Japan	South Korea	Envy-freeness
USA	0	− 195	− 163	116	1208	1447	1447	1447	1447	− 195
Russia	898	0	594	596	2090	2518	2518	2518	2518	0
Canada	299	− 128	0	− 260	1252	1537	1537	1537	1537	− 260
Denmark	856	194	444	0	370	1771	1771	1771	1771	0
Norway	849	45	431	− 28	0	1581	1581	1581	1581	− 28
Iceland	0	− 363	− 271	− 699	− 1274	0	0	0	0	− 1274
China	0	0	0	0	0	0	0	0	0	0
Japan	− 414	− 142	− 234	0	0	0	0	0	0	− 414
South Korea	− 10	0	0	0	0	0	0	0	0	− 10

Table 15 Satisfaction level of countries according to scenario 21

Country	Satisfaction level
USA	– 3494
Russia	– 3493
Canada	– 3494
Denmark	– 3495
Norway	– 3494
Iceland	– 2607
China	0
Japan	– 791
South Korea	– 10
Total	– 20,877

Denmark and Norway are the most unsatisfied countries, however, their satisfaction levels are almost equal to each other. Additionally, we provide information on the number of shared areas between two countries (see Table 16).

According to Table 16, most areas are shared by Denmark and other Arctic states such as Canada, Norway, Russia and the USA.

6 Conclusion

Using introduced model of utility values with respect to main resources—oil, gas, fish, as well as maritime routes—and different scenarios for an allocation of territories beyond EEZs among interested countries we evaluate dissatisfaction of countries using different models.

Two main approaches are used—each territory is allocated to a single country, and each territory can be allocated to two countries—so called shared allocation. There are two types of areas allocation models. The first type of models is based on the distance of areas to the country while the second type of models is based on the satisfaction level. It has been shown that first type models provide better results in terms of the total satisfaction level, however, the level of envy-freeness of these models is relatively high. The second type models provide slightly lower total satisfaction level. On the other hand, satisfaction levels are equal for most countries interested in the Arctic region which means that allocations based on these models are more fair.

Allocation with respect to the level of interest in the area (scenarios 1–2) allows to maximize the total satisfaction level but the difference in individual satisfaction levels can be extremely large. Allocations by scenarios 3–18 provide similar results (88.3% are allocated to the same country by all scenarios) with almost equal satisfaction level for each country. Note that superposition model (scenarios 15–18) allows to decrease the total number of exchange iterations. We may also consider the areas allocation problem as an integer linear programming problem (scenario 20), however, the results are slightly worse comparing to scenarios 3–18 since satisfaction levels of countries are not equal. In Sect. 4.2 we also provide a technique that combines these scenarios in

Table 16 Number of shared areas according to scenario 21

Countries	USA	Russia	Canada	Denmark	Norway	Iceland	China	Japan	South Korea
USA	0	0	1	351	1	0	0	0	0
Russia		0	0	385	0	0	0	0	0
Canada			0	911	0	0	0	0	0
Denmark				0	667	0	0	0	0
Norway					0	0	0	0	0
Iceland						0	0	0	0
China							0	0	0
Japan								0	0
South Korea									0

order to obtain more consistent results. We propose as well a modified adjusted winner procedure which provides similar satisfaction levels in terms of group interests.

We would like first to point out that we do not provide an allocation of territories in the Arctic region. We try to provide a tool to solve this problem. Several algorithms used for this aim allows us to compare their efficiency to resolve the stated problem.

There are several issues left for future research. First, in our work we consider four main resources, however, it will be useful to enrich data and add other types of countries' interests in the region. Second, we have evaluated the importance of each resource equally which is arguable and require a more detailed study. The importance of natural resources may also vary depending on the country. The presented models can be extended to the case of unequal importance of different resources which was the case in Demin and Shvydun (2017). Future development of the models can also include models with transferable utility and models where countries evaluate their utility or level of interests differently.

We should also note that an application of the model developed to disputable zones between Russia and Norway in Barents Sea shows almost ideal coincidence of the allocation of territories between these countries provided by the model and the allocation obtained in 2010 as a result of bilateral talks between these countries (Demin and Shvydun 2017). It is worth pointing out that this very surprising for us result was obtained due to the hard and efficient (almost 40 years long) work of diplomats of both sides.

We strongly believe that the provided model might ease such work for interested countries and lead to similar efficient allocations.

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